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HOW TO IMPLEMENT A SIMPLE ROUTING SYSTEM(U) GEORGIA
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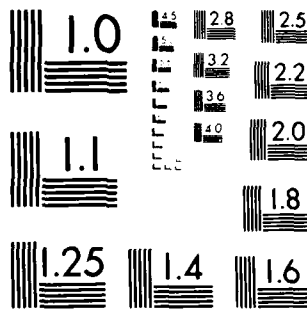
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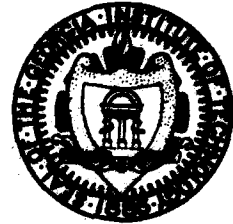
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HOW TO IMPLEMENT A SIMPLE ROUTING SYSTEM

BY

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PDRC 83-04

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This report

Summary

We explain how to implement a simple but effective routing system that will quickly determine (1) which locations should be visited by which delivery vehicles, and (2) in what sequence each vehicle should visit its assigned locations. The system uses only a map and two card files. It is easy to maintain and requires no additional clerical effort. The system is being used successfully by a large delivery organization in Atlanta.



Little office

I M P O R T A N T

To receive any revisions or additional information, please fill out
and mail this page. Thank you!

Your name _____

Your address _____

Name of meal program _____

Name of
sponsoring agency _____

Address of
sponsoring agency _____

Approximate number of
meals served daily _____

Please mail to: Professor John Bartholdi
 School of Industrial and Systems Engineering
 Georgia Tech
 Atlanta, Georgia 30332

How to Implement a Simple Routing System

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1. Introduction

This manual explains a very simple, yet effective method for managing large delivery systems. It was designed to help organizations where, without the help of a computer, someone must route delivery vehicles so that they visit many delivery locations as quickly as possible.

This system is intended for organizations that make regular deliveries, perhaps daily, to 40-500 various locations from a single depot. The list of clients may change over time, but not "too rapidly." (For example, this system will be helpful to newspaper, meal, or fuel oil delivery, but less helpful - in this form - for furniture delivery, where the clients are completely different each day.) The number of delivery vehicles may vary from day-to-day, and to further complicate matters, the list of clients served may change frequently, and so the delivery locations may also change frequently. Each delivery day the manager must prepare a list of locations to be visited. The locations must be divided into a number of routes equal to the number of delivery vehicles available that day. Because the deliveries should be made quickly, the sequence of visits within each route should be the most time-efficient. Also, the routes should take approximately the same time to complete so that the work is shared equitably among the drivers.

The task of designing these routes, already difficult, is all the more difficult because the details of the problem change so frequently. Any routes the manager constructs are soon obsolete. Such a problem could easily occupy the full-time services of a large computer. Our routing method will help solve this problem without a computer and even without any additional clerical effort. The simplicity and effectiveness of this method are based on new theoretical results by the authors at Georgia Institute of Technology (Appendix 2).

Knowing the delivery locations and the number of available delivery vehicles with our method, you can

- A. Determine which vehicles should visit what locations.
- B. Determine a suggested sequence in which the locations should be visited.
- C. Update A and B above when clients leave the system or join the system.

The routes generated in A and B above will ensure that all deliveries are made quickly and that the work is shared fairly among the vehicles. The generated routes will usually NOT be the very best possible, but to find the very best routes would require a large computer and specially trained people. Nevertheless, the routes generated by our method are guaranteed to be quite good on the average and well worth using.

2. The Routing System

The routing system consists of: an ALPHABETICAL FILE, a ROUTE FILE, a city map and a table of THETA numbers.

A. The ALPHABETICAL FILE is a flat card file, such as a Rolodex™, which contains a set of cards sorted alphabetically by client. Each card in this file lists a client's name, address, telephone number, and THETA number, together with any additional information related to delivery instructions or client needs (Figure 4).

B. The ROUTE FILE is a circular card file, such as a Rolodex™, which

contains a set of cards identical to those in the ALPHABETICAL FILE, but sorted numerically according to their THETA values, from smallest to largest (Figure 4).

C. The map is a street map of the delivery region. For ease of reading the delivery region should be at least one foot along each side and preferably larger (two to three feet along each side would be most convenient).

D. The table of THETA values is provided in Appendix 3.

3. Setting Up the Routing System

Step 1. On the map draw the smallest possible square (length = width) that contains all present and anticipated client locations. This square may be rotated in any direction in order to minimize its total area. Ideally, the resulting square will be two or three feet on each side. If the square is less than about one foot on each side, it is too small and you should use a larger map.

Step 2. Once the minimum area square has been constructed, divide each side into ten equal lengths to form a 10 x 10 grid. Label the divisions 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 (Figure 1). To help you read numbers within each division more accurately, draw small marks at equal intervals along the bottom and left edges of the grid (also shown in Figure 1).

Alternatively, for easier reading, you could draw the grid on a sheet of transparent plastic and attach this to the map. If you do this, make sure that the grid is firmly attached so that it does not move with respect to the map.

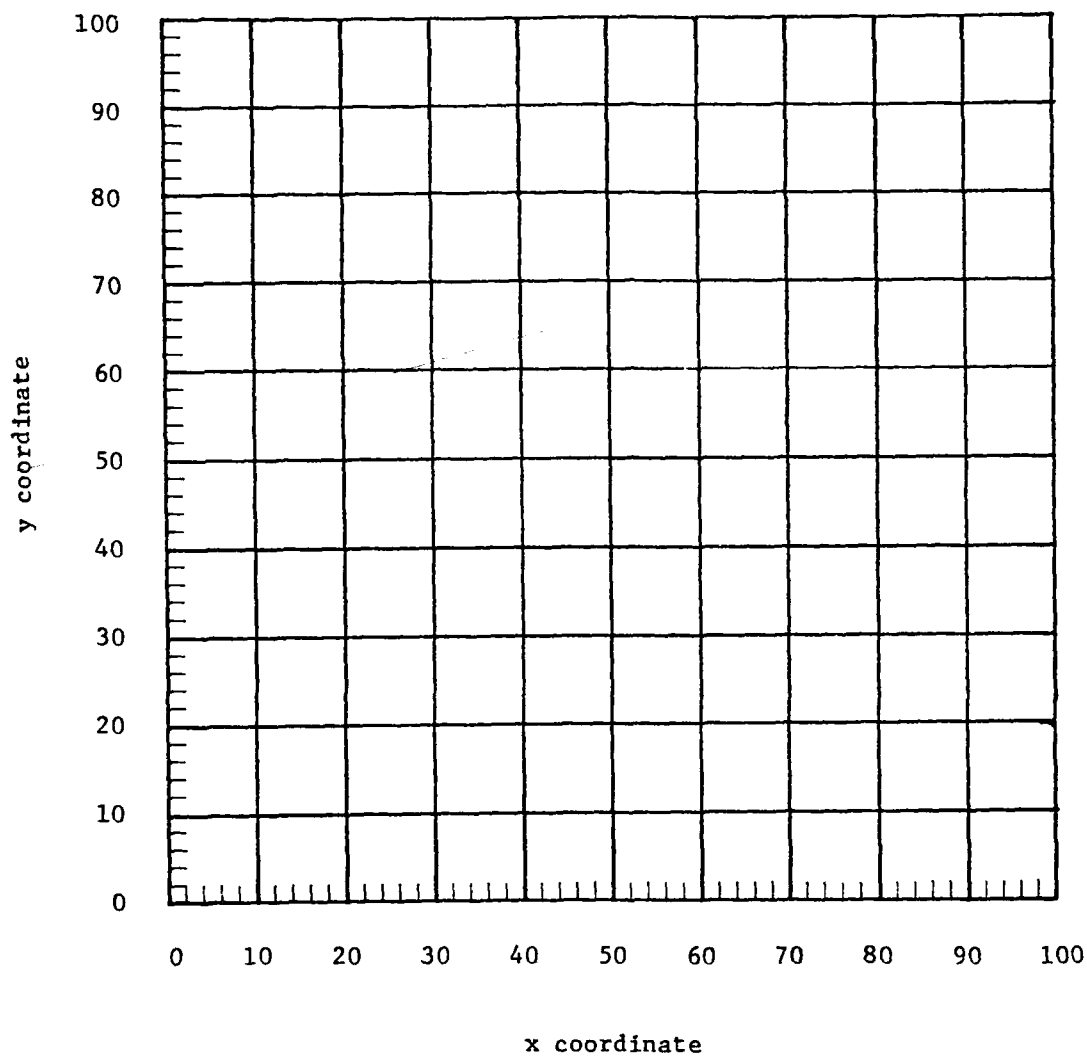


Figure 1: Prepare a grid of sufficient size to cover the distribution area. The grid may be drawn either directly on the map or else on a transparent plastic overlay. The grid should be divided so that you can read or estimate points between 0 and 100.

Step 3. Now build the ALPHABETICAL FILE and the ROUTE FILE.

(This step entails most of the effort necessary to set up the system. However, it needs to be done only once, and, afterward, requires very little effort to maintain.)

A. For each client prepare two identical cards with name, address, telephone number, and any other pertinent information.

B. Find client's location on the map and write the coordinates of the location. This must be done in the form (x,y) where x = the number of divisions from the left of grid, and y = the number of divisions from the bottom of grid (Figure 2). Remember to always write x first and then y.

C. Look in the Table of THETA numbers (Appendix 3) and read the THETA number that corresponds to the coordinates of the client location (Figure 3). Write this THETA on each of the client's two cards. File one card alphabetically by last name in the ALPHABETICAL FILE; file the other card by THETA value (smallest to largest) in the ROUTE FILE (Figure 4).

The ALPHABETICAL FILE enables you to keep track of people by name. The ROUTE FILE lists all clients in the order they should be visited; since this depends on their THETA numbers, it is critical that these be read accurately and filed in proper order!

Step 4. Partition the cards of the ROUTE FILE according to the number of delivery vehicles you have (for example, if you have 4 vehicles, then partition into four equal groups of cards). Each partition now lists all of the locations - in the proper sequence of deliveries - to be visited by one vehicle (Figure 5).

The partitioning of Step 4 may need to be adjusted through trial-and-error. If possible, you should partition where there are natural gaps in

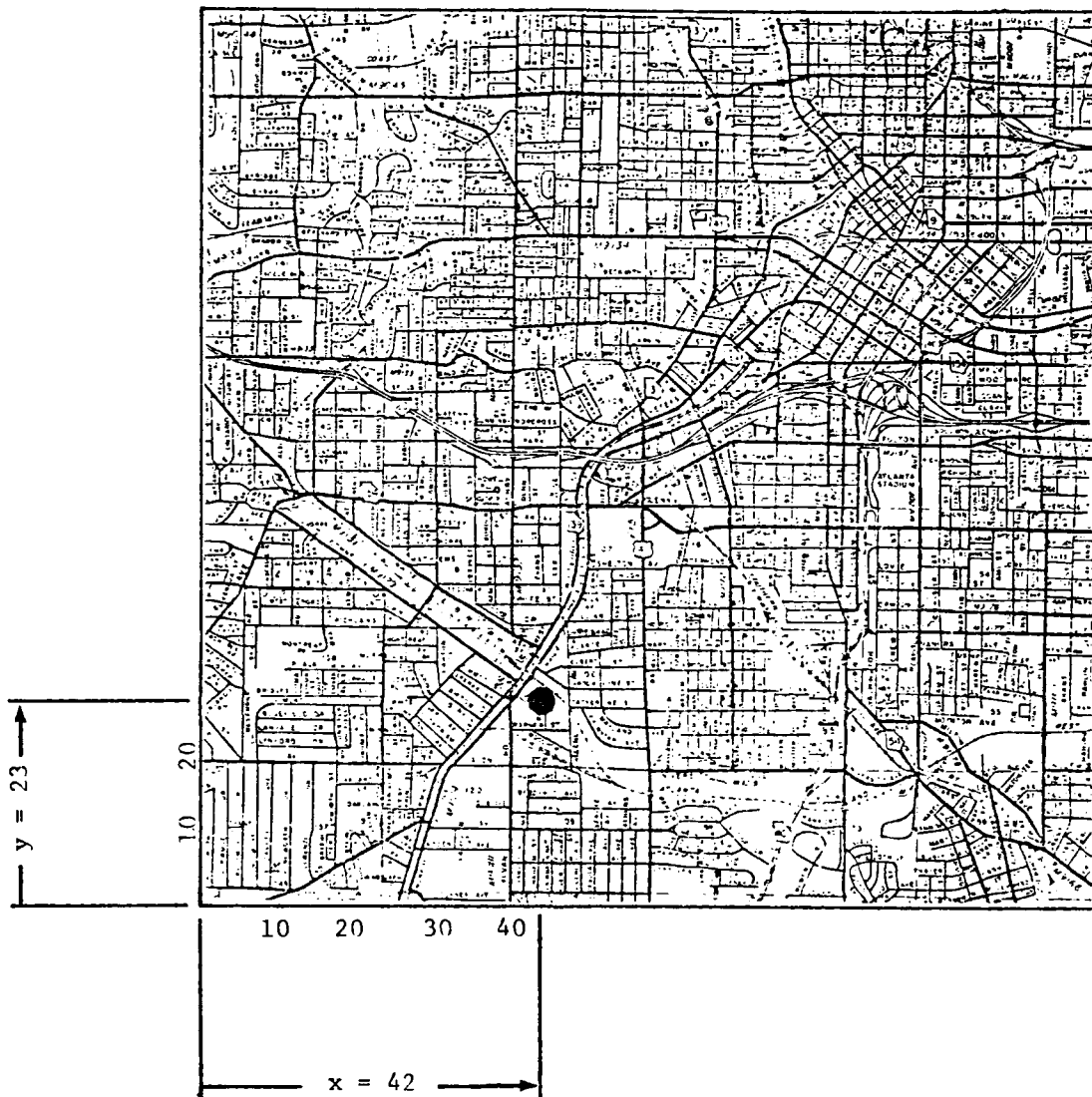


Figure 2: Using the grid of Figure 1, we can read the (x,y) coordinates of this example point. x is the number of divisions from the left and y is the number of divisions from the bottom.

				x ↓							
X=	40	41	(42)	43	44	45	46	47	48	49	
Y=											
0	941	940	940	940	938	938	938	937	937	937	
1	941	940	940	940	938	938	938	937	937	937	
2	940	940	940	940	938	938	938	937	937	937	
3	941	939	940	940	938	938	938	937	937	937	
4	942	942	939	939	939	939	935	936	936	936	
5	942	943	943	939	939	935	935	936	936	936	
6	942	943	943	943	935	935	935	936	936	936	
7	944	943	943	931	931	935	935	934	934	934	
8	943	943	931	931	931	932	935	934	934	934	
9	943	931	931	931	931	932	932	934	934	934	
10	930	931	930	930	932	932	933	933	933	933	
11	929	930	930	930	932	932	932	933	933	933	
12	929	930	930	930	932	932	932	933	933	933	
13	929	928	928	928	926	926	926	926	925	925	
14	929	929	928	928	926	926	926	926	925	925	
15	929	928	928	928	927	927	926	925	925	925	
16	915	928	928	927	927	927	924	925	925	925	
17	915	915	927	927	927	924	924	924	924	924	
18	915	915	915	927	923	923	924	924	924	924	
19	916	916	916	919	923	923	923	923	922	922	
20	916	916	919	919	919	923	923	923	922	922	
21	916	919	919	919	920	920	923	922	922	922	
22	918	919	919	919	920	920	920	922	922	922	
(23)	918	918	(919)	919	920	920	921	921	922	922	
24	918	918	918	918	920	921	921	921	921	921	
25	894	893	893	893	891	891	891	890	890	890	
26	894	893	893	893	891	891	891	890	890	890	
27	894	893	893	893	891	891	891	891	890	890	
28	895	893	892	892	892	892	892	890	889	889	
29	895	896	892	892	892	892	889	889	890	889	
30	895	896	896	892	892	888	888	889	889	889	
31	897	896	896	896	888	888	888	887	887	887	
32	897	896	896	884	884	888	888	888	887	887	
33	897	896	884	884	884	885	888	888	887	887	
34	897	884	884	884	885	885	885	887	887	887	
35	883	883	884	883	885	885	886	886	887	886	
36	883	883	883	883	885	885	886	886	886	886	
37	883	883	883	883	885	885	885	886	886	886	
38	882	882	881	881	879	879	879	879	879	878	
39	882	882	881	881	880	880	879	879	878	878	
40	882	881	881	881	880	880	880	878	878	878	
41	118	118	881	881	880	880	877	877	878	878	
42	118	118	119	880	880	877	877	877	877	878	
43	118	118	119	119	876	877	877	877	877	877	
44	119	119	119	123	876	876	876	876	876	875	
45	119	119	122	122	123	876	876	876	875	875	
46	119	122	122	122	123	123	876	875	875	875	
47	121	122	122	122	123	123	124	124	875	875	
48	121	121	122	122	123	124	124	124	124	875	
49	121	121	121	122	124	124	124	124	124	124	

Figure 3: This is an example of how to read the Table of THETA Numbers. To $x = 42$ and $y = 23$, we read the corresponding THETA = 919

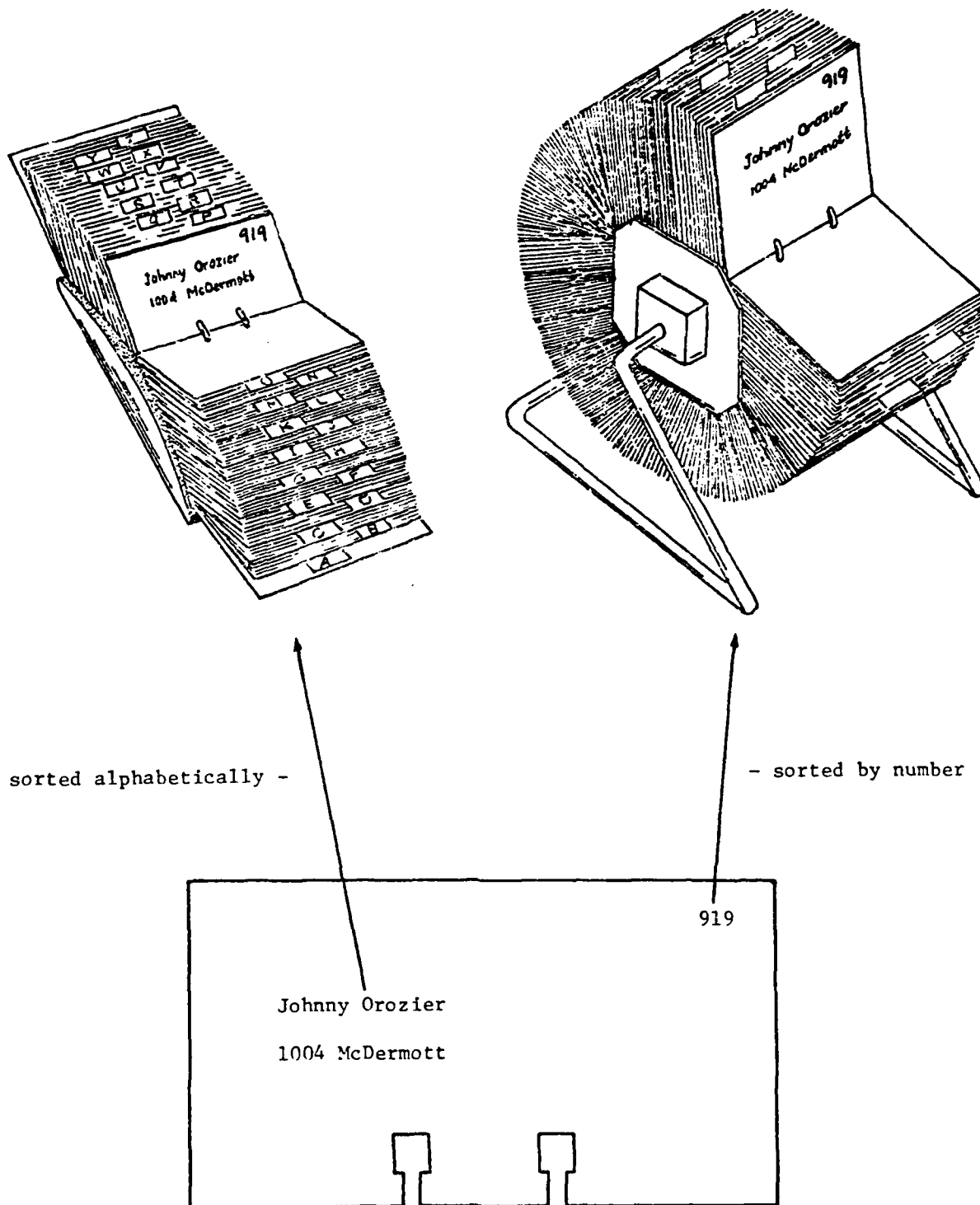


Figure 4: The ALPHABETICAL FILE and the ROUTE FILE.

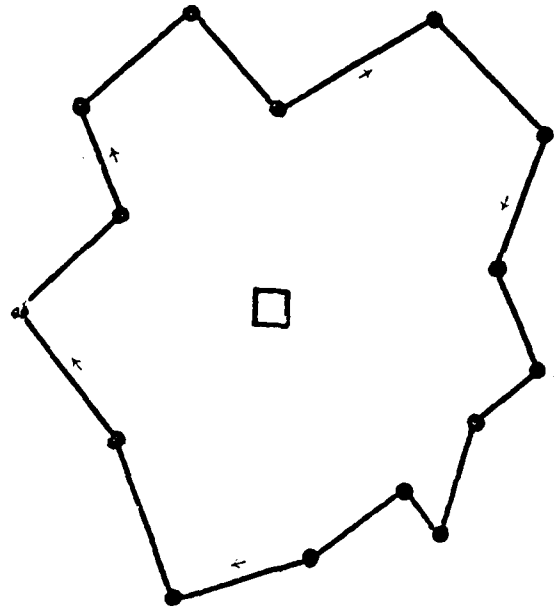
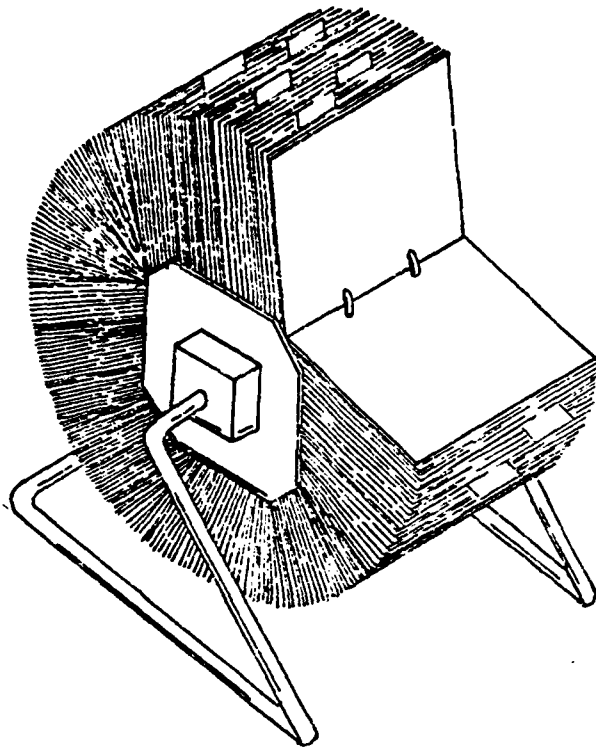


Figure 5A: The sorted cards give an efficient route for one driver.

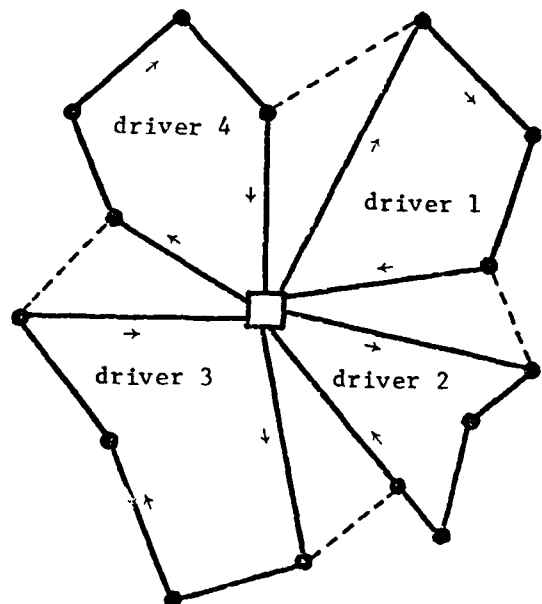
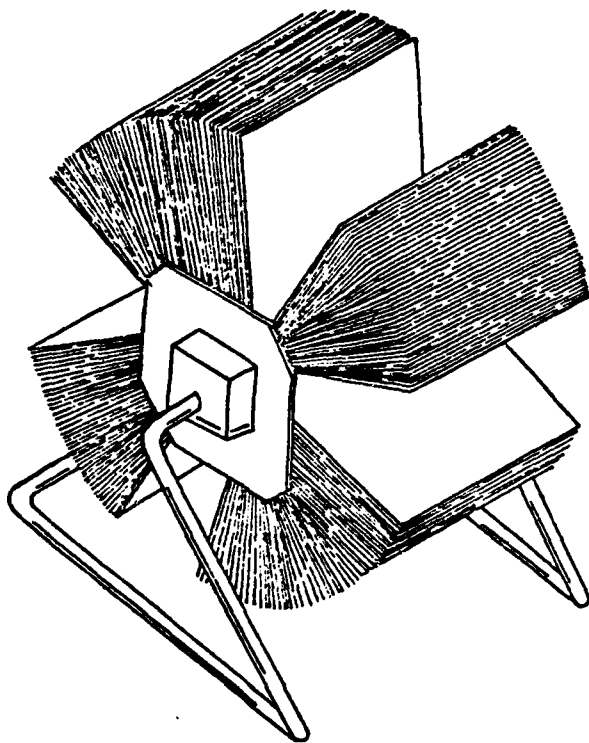


Figure 5B: Partitioning the cards give efficient routes for more than one driver. The cards within each group determine a route.

the delivery locations and so avoid separating a cluster of nearby locations into different routes.

We recommend that you partition the cards equally at first, and then time the drivers on these routes. If some routes are too long, change the partition so that these routes have fewer cards. When you have a satisfactory partition, mark it in the ROUTE FILE by tabs.

4. Maintaining the System

A major advantage of our system is the ease with which you can update your delivery plans to account for changes.

A. To remove a client:

- (1) Find the client's card in the ALPHABETICAL FILE; remove the card and note client's THETA.
- (2) Using the client's THETA number, find the client's other card in the ROUTE FILE and remove it.

B. To add a client

- (1) Find the client's location on the map and read the (x,y) coordinates (that is, x divisions from the left and y divisions from the bottom of the grid).
- (2) Look in the Table of THETA numbers in Appendix 3 to find the THETA that corresponds to this (x,y).
- (3) Prepare two identical cards, each one listing the client's name, address, telephone number, miscellaneous information, and THETA.

- (4) File one card alphabetically by the client's last name in the ALPHABETICAL FILE; file the other card by the client's THETA number (smallest to largest) in the ROUTE FILE.

Addition and removal of clients should be done daily. Eventually the cumulative effect of these daily changes will make it necessary to adjust the route partitions. By timing the routes you can decide when repartitioning is necessary. (Generally, repartitioning needs to be done every two to four months.)

There may be temptation to add new clients to routes without using the THETA Table. We caution you against this. Each time you do this, a small, perhaps unnoticeable error is introduced. After several months the errors will have accumulated so that the routes will have become much too long. If the THETA's are not used properly, the system will slowly degenerate until the system has lost its efficiency.

APPENDIX 1

Questions. (Please telephone or write with additional questions.)

Will this help everyone?

No. If you are servicing less than 20-30 delivery locations, you can probably do as well by "eyeball." Our method will be most useful for larger delivery systems.

Also, our method is designed for urban or suburban delivery systems. We do not expect it to be as useful in rural areas where the road network is more sparse; in such situations it is again possible to do well by "eyeball."

How long will it take to build this routing system?

One to five days; the bulk of the work is in preparing cards for the clients already being served, and drawing the grid for the map.

Where can I get the materials?

<u>Items</u>	<u>Typical Cost</u>	<u>Available from</u>
° round card file (e.g. Rolodex™)	\$25.00	office supply store
° flat card file (e.g. Rolodex™)	\$20.00	office supply store
° Street maps	\$ 5.00	Department of Transportation of your state.

- | | | |
|--|-----------------------------|-----------------------------------|
| ° map mounts, including backing, plexiglass cover, and clips | \$15-\$35 depending on size | art supply store or frame shop |
| ° fine-tipped pens for drawing grid on plastic | \$8.00 per set | art supply or office supply store |

What if the suggested sequence of locations to be visited is obviously not best?

Sometimes this will happen. There are two ways to handle it. One is to ignore it since it will happen infrequently and will generally not make much difference (that is, the route will not be much longer than the optimal route). The other is to change the suggested sequence of deliveries appropriately. We recommend that this be done only with the manager's approval, since, in our experience, drivers have frequently thought that they could improve the route but were mistaken. Unless you are absolutely convinced of an improvement, you should visit the delivery locations according to their THETA numbers.

Incidentally, the routes generated by our method typically have a relatively long travel time to the first and from the last location. Again, this tends to make little difference and can generally be ignored.

What if the central depot is far away from the delivery region?

It does not matter. The grid should be centered over the delivery region, but the depot does not have to be in the center of the grid. In fact it does not have to be within the grid at all.

What if there is a river running through the delivery region?

In this case the system might not work well. Our method is based on the assumption that if two locations are close together on the map, then

it is relatively quick to travel from one to the other. If a river divides the delivery region, this might not be true since locations on opposite sides of the river might be far apart by car. One way of handling this is to implement a different routing system for each side of the river.

APPENDIX 2

Why the System Works

This section is just for the curious; you certainly do not have to understand mathematics to use the system, even though the method is based on a mathematical idea, that of a "spacefilling curve". A "spacefilling curve" is an infinitely crinkly version of the pattern shown in Figure 5; it may be imagined to be the route of an incredibly industrious driver who visits every single point on the map. Our method is to visit locations in the same sequence as the "spacefilling curve", but to travel directly between them, as shown in Figure 5. The THETA numbers represent the relative position of locations along the "curve," so when you sort the cards by THETA's, you are sequencing locations as they are visited by the "spacefilling curve," just as in Figure 6.

Technical details may be found in the following references.

1. Bartholdi, J. and L. Platzman (1982). "An $O(n \log n)$ heuristic for the planar travelling salesman problem based on spacefilling curves," Operations Research Letters 1 ():
2. Platzman, L. and J. Bartholdi (1983). "Space-filling curves and routing problems in the plane," PDRC Report 83-01, Georgia Institute of Technology, submitted to Mathematics of Operations Research.

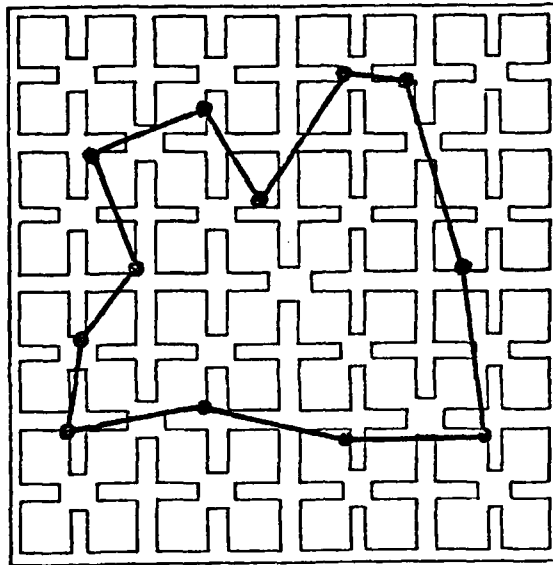


Figure 6: Our method visits locations according to the sequence in which they appear along a "spacefilling curve". The THETA number of a location gives its relative position along the curve.

APPENDIX 3

Table of THETA numbers

Immediately after this page is the Table of THETA numbers. Remove these pages, fold in half like a booklet (making sure the cover is on the outside), and staple the spine.

TABLE OF THETA NUMBERS

For instructions on how to use this table,
see J. Bartholdi, et al., (April 1983), "How
to Implement a Simple Routing System," Georgia
Institute of Technology, School of Industrial
and Systems Engineering.

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	90	91	92	93	94	95	96	97	98	99	Σ
565	565	565	565	563	563	563	562	562	562	562	50
565	565	565	565	563	563	563	562	562	562	562	51
565	565	564	564	564	564	561	561	561	561	561	52
567	567	564	564	564	564	561	561	561	561	561	53
568	568	568	568	566	568	561	561	561	561	561	54
568	568	568	568	566	568	561	561	561	561	561	55
568	568	566	566	566	566	560	560	560	560	560	56
568	568	566	566	566	566	560	560	560	560	560	57
568	568	566	566	566	566	560	560	560	560	560	58
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568	568	566	566	566	566	560	560	560	560	560	60
568	568	566	566	566	566	560	560	560	560	560	61
568	568	566	566	566	566	560	560	560	560	560	62
568	568	566	566	566	566	560	560	560	560	560	63
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568	568	566	566	566	566	560	560	560	560	560	66
568	568	566	566	566	566	560	560	560	560	560	67
568	568	566	566	566	566	560	560	560	560	560	68
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568	568	566	566	566	566	560	560	560	560	560	81
568	568	566	566	566	566	560	560	560	560	560	82
568	568	566	566	566	566	560	560	560	560	560	83
568	568	566	566	566	566	560	560	560	560	560	84
568	568	566	566	566	566	560	560	560	560	560	85
568	568	566	566	566	566	560	560	560	560	560	86
568	568	566	566	566	566	560	560	560	560	560	87
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493	506	506	505	505	505	505	502	502	503	503	91
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196	496	496	497	499	499	499	499	499	499	499	9

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Y=										
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7	756	756	755	744	747	747	747	747	746	746
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11	742	742	743	745	744	745	745	745	746	746
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13	741	741	741	739	739	738	738	738	738	738
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9	770	772	772	772	772	758	757	757	742	742	60
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19	779	724	724	724	723	723	723	729	729	729	70
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47	48	183	183	183	175	175	175	175	174	174	172	144	145
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8	9	797	797	797	796	796	765	765	765	764	764	36	37
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	60	61	62	63	64	65	66	67	68	69	Σ	\bar{X}
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